

**Corrigé de l'exercice 1**Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$\begin{aligned} P(x) &= x^2 - 18x - 7 \\ &= (x - 9)^2 - 9^2 - 7 \\ &= (x - 9)^2 - 81 - 7 \end{aligned}$$

$$\boxed{P(x) = (x - 9)^2 - 88}$$

$$\begin{aligned} R(x) &= 64x^2 - 16x + 1 \\ &= (8x - 1)^2 \\ &= \left(8 \times \left(x - \frac{1}{8}\right)\right)^2 \end{aligned}$$

$$\boxed{R(x) = 64 \times \left(x - \frac{1}{8}\right)^2}$$

$$\begin{aligned} Q(x) &= 5x^2 + 5x - 5 \\ &= 5 \times (x^2 + x - 1) \\ &= 5 \times \left( \left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 1 \right) \\ &= 5 \times \left( \left(x + \frac{1}{2}\right)^2 + \frac{-1}{4} - \frac{1 \times 4}{1 \times 4} \right) \\ &= 5 \times \left( \left(x + \frac{1}{2}\right)^2 + \frac{-1}{4} - \frac{4}{4} \right) \\ &= 5 \times \left( \left(x + \frac{1}{2}\right)^2 + \frac{-5}{4} \right) \end{aligned}$$

$$\boxed{Q(x) = 5 \times \left(x + \frac{1}{2}\right)^2 + \frac{-25}{4}}$$

$$\begin{aligned} S(x) &= x^2 - 3x - 5 \\ &= \left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 5 \\ &= \left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{5 \times 4}{1 \times 4} \\ &= \left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{20}{4} \end{aligned}$$

$$\boxed{S(x) = \left(x - \frac{3}{2}\right)^2 + \frac{-29}{4}}$$

**Corrigé de l'exercice 2**Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$\begin{aligned} P(x) &= x^2 + 9x - 3 \\ &= \left(x + \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 - 3 \\ &= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} - \frac{3 \times 4}{1 \times 4} \\ &= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} - \frac{12}{4} \end{aligned}$$

$$\boxed{P(x) = \left(x + \frac{9}{2}\right)^2 + \frac{-93}{4}}$$

$$\begin{aligned} Q(x) &= 5x^2 + 4x + 6 \\ &= 5 \times \left(x^2 + \frac{4}{5}x + \frac{6}{5}\right) \\ &= 5 \times \left( \left(x + \frac{2}{5}\right)^2 - \left(\frac{2}{5}\right)^2 + \frac{6}{5} \right) \\ &= 5 \times \left( \left(x + \frac{2}{5}\right)^2 + \frac{-4}{25} + \frac{6 \times 5}{5 \times 5} \right) \\ &= 5 \times \left( \left(x + \frac{2}{5}\right)^2 + \frac{-4}{25} + \frac{30}{25} \right) \\ &= 5 \times \left( \left(x + \frac{2}{5}\right)^2 + \frac{26}{25} \right) \\ &= 5 \times \left(x + \frac{2}{5}\right)^2 + \frac{26 \times 5}{5 \times 5} \end{aligned}$$

$$\boxed{Q(x) = 5 \times \left(x + \frac{2}{5}\right)^2 + \frac{26}{5}}$$

$$\begin{aligned} R(x) &= x^2 + 4x - 6 \\ &= (x + 2)^2 - 2^2 - 6 \\ &= (x + 2)^2 - 4 - 6 \end{aligned}$$

$$\boxed{R(x) = (x + 2)^2 - 10}$$

$$\begin{aligned} S(x) &= 4x^2 - 24x + 36 \\ &= (2x - 6)^2 \\ &= \left(2 \times \left(x - \frac{6}{2}\right)\right)^2 \end{aligned}$$

$$\boxed{S(x) = 4 \times \left(x - 3\right)^2}$$

**Corrigé de l'exercice 3**

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = -5x^2 + 4x + 1$$

$$= -5 \times \left( x^2 - \frac{4}{5}x - \frac{1}{5} \right)$$

$$= -5 \times \left( \left( x - \frac{2}{5} \right)^2 - \left( \frac{2}{5} \right)^2 + \frac{-1}{5} \right)$$

$$= -5 \times \left( \left( x - \frac{2}{5} \right)^2 + \frac{-4}{25} + \frac{-1 \times 5}{5 \times 5} \right)$$

$$= -5 \times \left( \left( x - \frac{2}{5} \right)^2 + \frac{-4}{25} + \frac{-5}{25} \right)$$

$$= -5 \times \left( \left( x - \frac{2}{5} \right)^2 + \frac{-9}{25} \right)$$

$$= -5 \times \left( x - \frac{2}{5} \right)^2 + \frac{-9 \times 5 \times (-1)}{5 \times 5}$$

$$P(x) = -5 \times \left( x - \frac{2}{5} \right)^2 + \frac{9}{5}$$

$$Q(x) = 16x^2 - 8x + 1$$

$$= (4x - 1)^2$$

$$= \left( 4 \times \left( x - \frac{1}{4} \right) \right)^2$$

$$Q(x) = 16 \times \left( x - \frac{1}{4} \right)^2$$

$$R(x) = x^2 + 6x - 6$$

$$= (x + 3)^2 - 3^2 - 6$$

$$= (x + 3)^2 - 9 - 6$$

$$R(x) = (x + 3)^2 - 15$$

$$S(x) = x^2 + 5x + 7$$

$$= \left( x + \frac{5}{2} \right)^2 - \left( \frac{5}{2} \right)^2 + 7$$

$$= \left( x + \frac{5}{2} \right)^2 + \frac{-25}{4} + \frac{7 \times 4}{1 \times 4}$$

$$= \left( x + \frac{5}{2} \right)^2 + \frac{-25}{4} + \frac{28}{4}$$

$$S(x) = \left( x + \frac{5}{2} \right)^2 + \frac{3}{4}$$

### Corrigé de l'exercice 4

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = x^2 - 7x + 4$$

$$= \left( x - \frac{7}{2} \right)^2 - \left( \frac{7}{2} \right)^2 + 4$$

$$= \left( x - \frac{7}{2} \right)^2 + \frac{-49}{4} + \frac{4 \times 4}{1 \times 4}$$

$$= \left( x - \frac{7}{2} \right)^2 + \frac{-49}{4} + \frac{16}{4}$$

$$P(x) = \left( x - \frac{7}{2} \right)^2 + \frac{-33}{4}$$

$$Q(x) = 16x^2 - 64x + 64$$

$$= (4x - 8)^2$$

$$= \left( 4 \times \left( x - \frac{8}{4} \right) \right)^2$$

$$Q(x) = 16 \times (x - 2)^2$$

$$S(x) = x^2 - 4x - 1$$

$$= (x - 2)^2 - 2^2 - 1$$

$$= (x - 2)^2 - 4 - 1$$

$$S(x) = (x - 2)^2 - 5$$

$$R(x) = 3x^2 + 4x + 6$$

$$= 3 \times \left( x^2 + \frac{4}{3}x + 2 \right)$$

$$= 3 \times \left( \left( x + \frac{2}{3} \right)^2 - \left( \frac{2}{3} \right)^2 + 2 \right)$$

$$= 3 \times \left( \left( x + \frac{2}{3} \right)^2 + \frac{-4}{9} + \frac{2 \times 9}{1 \times 9} \right)$$

$$= 3 \times \left( \left( x + \frac{2}{3} \right)^2 + \frac{-4}{9} + \frac{18}{9} \right)$$

$$= 3 \times \left( \left( x + \frac{2}{3} \right)^2 + \frac{14}{9} \right)$$

$$= 3 \times \left( x + \frac{2}{3} \right)^2 + \frac{14 \times 3}{3 \times 3}$$

$$R(x) = 3 \times \left( x + \frac{2}{3} \right)^2 + \frac{14}{3}$$

### Corrigé de l'exercice 5

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = 5x^2 + 3x - 9$$

$$= 5 \times \left( x^2 + \frac{3}{5}x - \frac{9}{5} \right)$$

$$= 5 \times \left( \left( x + \frac{3}{10} \right)^2 - \left( \frac{3}{10} \right)^2 + \frac{-9}{5} \right)$$

$$= 5 \times \left( \left( x + \frac{3}{10} \right)^2 + \frac{-9}{100} + \frac{-9 \times 20}{5 \times 20} \right)$$

$$= 5 \times \left( \left( x + \frac{3}{10} \right)^2 + \frac{-9}{100} + \frac{-180}{100} \right)$$

$$= 5 \times \left( \left( x + \frac{3}{10} \right)^2 + \frac{-189}{100} \right)$$

$$= 5 \times \left( x + \frac{3}{10} \right)^2 + \frac{-189 \times 5}{5 \times 20}$$

$$P(x) = 5 \times \left( x + \frac{3}{10} \right)^2 + \frac{-189}{20}$$

$$Q(x) = x^2 - 3x - 2$$

$$= \left( x - \frac{3}{2} \right)^2 - \left( \frac{3}{2} \right)^2 - 2$$

$$= \left( x - \frac{3}{2} \right)^2 + \frac{-9}{4} - \frac{2 \times 4}{1 \times 4}$$

$$= \left( x - \frac{3}{2} \right)^2 + \frac{-9}{4} - \frac{8}{4}$$

$$Q(x) = \left( x - \frac{3}{2} \right)^2 + \frac{-17}{4}$$

$$R(x) = x^2 - 16x + 64$$

$$R(x) = (x - 8)^2$$

$$S(x) = x^2 + 8x + 9$$

$$= (x + 4)^2 - 4^2 + 9$$

$$= (x + 4)^2 - 16 + 9$$

$$S(x) = (x + 4)^2 - 7$$

### Corrigé de l'exercice 6

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = x^2 - 16x + 6$$

$$= (x - 8)^2 - 8^2 + 6$$

$$= (x - 8)^2 - 64 + 6$$

$$P(x) = (x - 8)^2 - 58$$

$$Q(x) = x^2 + 5x - 4$$

$$= \left( x + \frac{5}{2} \right)^2 - \left( \frac{5}{2} \right)^2 - 4$$

$$= \left( x + \frac{5}{2} \right)^2 + \frac{-25}{4} - \frac{4 \times 4}{1 \times 4}$$

$$= \left( x + \frac{5}{2} \right)^2 + \frac{-25}{4} - \frac{16}{4}$$

$$Q(x) = \left( x + \frac{5}{2} \right)^2 + \frac{-41}{4}$$

$$R(x) = 2x^2 + 8x + 2$$

$$= 2 \times (x^2 + 4x + 1)$$

$$= 2 \times ((x + 2)^2 - 2^2 + 1)$$

$$= 2 \times ((x + 2)^2 - 4 + 1)$$

$$= 2 \times ((x + 2)^2 - 3)$$

$$R(x) = 2 \times (x + 2)^2 - 6$$

$$S(x) = 9x^2 - 24x + 16$$

$$= (3x - 4)^2$$

$$= \left( 3 \times \left( x - \frac{4}{3} \right) \right)^2$$

$$S(x) = 9 \times \left( x - \frac{4}{3} \right)^2$$

### Corrigé de l'exercice 7

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = 25x^2 - 40x + 16$$

$$= (5x - 4)^2$$

$$= \left(5 \times \left(x - \frac{4}{5}\right)\right)^2$$

$$\boxed{P(x) = 25 \times \left(x - \frac{4}{5}\right)^2}$$

$$Q(x) = x^2 + 2x + 5$$

$$= (x + 1)^2 - 1^2 + 5$$

$$= (x + 1)^2 - 1 + 5$$

$$\boxed{Q(x) = (x + 1)^2 + 4}$$

$$R(x) = 4x^2 + 4x + 1$$

$$= 4 \times \left(x^2 + x + \frac{1}{4}\right)$$

$$= 4 \times \left(\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + \frac{1}{4}\right)$$

$$= 4 \times \left(\left(x + \frac{1}{2}\right)^2 + 0\right)$$

$$\boxed{R(x) = 4 \times \left(x + \frac{1}{2}\right)^2 + 0}$$

$$S(x) = x^2 + 3x - 1$$

$$= \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 1$$

$$= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{1 \times 4}{1 \times 4}$$

$$= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{4}{4}$$

$$\boxed{S(x) = \left(x + \frac{3}{2}\right)^2 + \frac{-13}{4}}$$

### Corrigé de l'exercice 8

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = 9x^2 - 30x + 25$$

$$= (3x - 5)^2$$

$$= \left(3 \times \left(x - \frac{5}{3}\right)\right)^2$$

$$\boxed{P(x) = 9 \times \left(x - \frac{5}{3}\right)^2}$$

$$R(x) = x^2 + 14x - 2$$

$$= (x + 7)^2 - 7^2 - 2$$

$$= (x + 7)^2 - 49 - 2$$

$$\boxed{R(x) = (x + 7)^2 - 51}$$

$$Q(x) = x^2 + 3x - 4$$

$$= \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 4$$

$$= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{4 \times 4}{1 \times 4}$$

$$= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{16}{4}$$

$$\boxed{Q(x) = \left(x + \frac{3}{2}\right)^2 + \frac{-25}{4}}$$

$$S(x) = 2x^2 + 5x + 6$$

$$= 2 \times \left(x^2 + \frac{5}{2}x + 3\right)$$

$$= 2 \times \left(\left(x + \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2 + 3\right)$$

$$= 2 \times \left(\left(x + \frac{5}{4}\right)^2 + \frac{-25}{16} + \frac{3 \times 16}{1 \times 16}\right)$$

$$= 2 \times \left(\left(x + \frac{5}{4}\right)^2 + \frac{-25}{16} + \frac{48}{16}\right)$$

$$= 2 \times \left(\left(x + \frac{5}{4}\right)^2 + \frac{23}{16}\right)$$

$$= 2 \times \left(x + \frac{5}{4}\right)^2 + \frac{23 \times 2}{2 \times 8}$$

$$\boxed{S(x) = 2 \times \left(x + \frac{5}{4}\right)^2 + \frac{23}{8}}$$