

**Corrigé de l'exercice 1**Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = -4x^2 - 8x + 8$$

$$= -4 \times (x^2 + 2x - 2)$$

$$= -4 \times ((x+1)^2 - 1^2 - 2)$$

$$= -4 \times ((x+1)^2 - 1 - 2)$$

$$= -4 \times ((x+1)^2 - 3)$$

$$\boxed{P(x) = -4 \times (x+1)^2 + 12}$$

$$Q(x) = x^2 + 11x + 8$$

$$= \left(x + \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 + 8$$

$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{8 \times 4}{1 \times 4}$$

$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{32}{4}$$

$$\boxed{Q(x) = \left(x + \frac{11}{2}\right)^2 + \frac{-89}{4}}$$

$$R(x) = 64x^2 - 64x + 16$$

$$= (8x - 4)^2$$

$$= \left(8 \times \left(x - \frac{4}{8}\right)\right)^2$$

$$\boxed{R(x) = 64 \times \left(x - \frac{1}{2}\right)^2}$$

$$S(x) = x^2 - 14x + 6$$

$$= (x - 7)^2 - 7^2 + 6$$

$$= (x - 7)^2 - 49 + 6$$

$$\boxed{S(x) = (x - 7)^2 - 43}$$

**Corrigé de l'exercice 2**Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = 9x^2 - 36x + 36$$

$$= (3x - 6)^2$$

$$= \left(3 \times \left(x - \frac{6}{3}\right)\right)^2$$

$$\boxed{P(x) = 9 \times (x - 2)^2}$$

$$Q(x) = x^2 + 11x + 5$$

$$= \left(x + \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 + 5$$

$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{5 \times 4}{1 \times 4}$$

$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{20}{4}$$

$$\boxed{Q(x) = \left(x + \frac{11}{2}\right)^2 + \frac{-101}{4}}$$

$$R(x) = x^2 - 6x + 8$$

$$= (x - 3)^2 - 3^2 + 8$$

$$= (x - 3)^2 - 9 + 8$$

$$\boxed{R(x) = (x - 3)^2 - 1}$$

$$S(x) = 4x^2 - 8x + 9$$

$$= 4 \times \left(x^2 - 2x + \frac{9}{4}\right)$$

$$= 4 \times \left((x - 1)^2 - 1^2 + \frac{9}{4}\right)$$

$$= 4 \times \left((x - 1)^2 - 1 + \frac{9}{4}\right)$$

$$= 4 \times \left((x - 1)^2 + \frac{-1 \times 4}{1 \times 4} + \frac{9}{4}\right)$$

$$= 4 \times \left((x - 1)^2 + \frac{-4}{4} + \frac{9}{4}\right)$$

$$= 4 \times \left((x - 1)^2 + \frac{5}{4}\right)$$

$$= 4 \times (x - 1)^2 + \frac{5 \times 4}{4 \times 1}$$

$$\boxed{S(x) = 4 \times (x - 1)^2 + 5}$$

**Corrigé de l'exercice 3**

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = x^2 + 9x + 4$$

$$= \left(x + \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 + 4$$

$$= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} + \frac{4 \times 4}{1 \times 4}$$

$$= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} + \frac{16}{4}$$

$$\boxed{P(x) = \left(x + \frac{9}{2}\right)^2 + \frac{-65}{4}}$$

$$Q(x) = x^2 - 2x + 1$$

$$= (x - 1)^2 - 1^2 + 1$$

$$= (x - 1)^2 - 1 + 1$$

$$\boxed{Q(x) = (x - 1)^2 + 0}$$

$$S(x) = 25x^2 + 50x + 25$$

$$= (5x + 5)^2$$

$$= \left(5 \times \left(x + \frac{5}{5}\right)\right)^2$$

$$\boxed{S(x) = 25 \times (x + 1)^2}$$

$$R(x) = 3x^2 - x + 4$$

$$= 3 \times \left(x^2 - \frac{1}{3}x + \frac{4}{3}\right)$$

$$= 3 \times \left(\left(x - \frac{1}{6}\right)^2 - \left(\frac{1}{6}\right)^2 + \frac{4}{3}\right)$$

$$= 3 \times \left(\left(x - \frac{1}{6}\right)^2 + \frac{-1}{36} + \frac{4 \times 12}{3 \times 12}\right)$$

$$= 3 \times \left(\left(x - \frac{1}{6}\right)^2 + \frac{-1}{36} + \frac{48}{36}\right)$$

$$= 3 \times \left(\left(x - \frac{1}{6}\right)^2 + \frac{47}{36}\right)$$

$$= 3 \times \left(x - \frac{1}{6}\right)^2 + \frac{47 \times 3}{3 \times 12}$$

$$\boxed{R(x) = 3 \times \left(x - \frac{1}{6}\right)^2 + \frac{47}{12}}$$

### Corrigé de l'exercice 4

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = 4x^2 + 16x + 16$$

$$= (2x + 4)^2$$

$$= \left(2 \times \left(x + \frac{4}{2}\right)\right)^2$$

$$\boxed{P(x) = 4 \times (x + 2)^2}$$

$$Q(x) = x^2 + 7x + 6$$

$$= \left(x + \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 6$$

$$= \left(x + \frac{7}{2}\right)^2 + \frac{-49}{4} + \frac{6 \times 4}{1 \times 4}$$

$$= \left(x + \frac{7}{2}\right)^2 + \frac{-49}{4} + \frac{24}{4}$$

$$\boxed{Q(x) = \left(x + \frac{7}{2}\right)^2 + \frac{-25}{4}}$$

$$R(x) = x^2 - 2x + 7$$

$$= (x - 1)^2 - 1^2 + 7$$

$$= (x - 1)^2 - 1 + 7$$

$$\boxed{R(x) = (x - 1)^2 + 6}$$

$$S(x) = 4x^2 - 3x + 8$$

$$= 4 \times \left(x^2 - \frac{3}{4}x + 2\right)$$

$$= 4 \times \left(\left(x - \frac{3}{8}\right)^2 - \left(\frac{3}{8}\right)^2 + 2\right)$$

$$= 4 \times \left(\left(x - \frac{3}{8}\right)^2 + \frac{-9}{64} + \frac{2 \times 64}{1 \times 64}\right)$$

$$= 4 \times \left(\left(x - \frac{3}{8}\right)^2 + \frac{-9}{64} + \frac{128}{64}\right)$$

$$= 4 \times \left(\left(x - \frac{3}{8}\right)^2 + \frac{119}{64}\right)$$

$$= 4 \times \left(x - \frac{3}{8}\right)^2 + \frac{119 \times 4}{4 \times 16}$$

$$\boxed{S(x) = 4 \times \left(x - \frac{3}{8}\right)^2 + \frac{119}{16}}$$

### Corrigé de l'exercice 5

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = -3x^2 - 3x + 8$$

$$= -3 \times \left( x^2 + x - \frac{8}{3} \right)$$

$$= -3 \times \left( \left( x + \frac{1}{2} \right)^2 - \left( \frac{1}{2} \right)^2 + \frac{-8}{3} \right)$$

$$= -3 \times \left( \left( x + \frac{1}{2} \right)^2 + \frac{-1 \times 3}{4 \times 3} + \frac{-8 \times 4}{3 \times 4} \right)$$

$$= -3 \times \left( \left( x + \frac{1}{2} \right)^2 + \frac{-3}{12} + \frac{-32}{12} \right)$$

$$= -3 \times \left( \left( x + \frac{1}{2} \right)^2 + \frac{-35}{12} \right)$$

$$= -3 \times \left( x + \frac{1}{2} \right)^2 + \frac{-35 \times 3 \times (-1)}{3 \times 4}$$

$$P(x) = -3 \times \left( x + \frac{1}{2} \right)^2 + \frac{35}{4}$$

$$Q(x) = x^2 - 12x + 4$$

$$= (x - 6)^2 - 6^2 + 4$$

$$= (x - 6)^2 - 36 + 4$$

$$Q(x) = (x - 6)^2 - 32$$

$$S(x) = 9x^2 - 54x + 81$$

$$= (3x - 9)^2$$

$$= \left( 3 \times \left( x - \frac{9}{3} \right) \right)^2$$

$$S(x) = 9 \times \left( x - 3 \right)^2$$

$$R(x) = x^2 + 5x - 5$$

$$= \left( x + \frac{5}{2} \right)^2 - \left( \frac{5}{2} \right)^2 - 5$$

$$= \left( x + \frac{5}{2} \right)^2 + \frac{-25}{4} - \frac{5 \times 4}{1 \times 4}$$

$$= \left( x + \frac{5}{2} \right)^2 + \frac{-25}{4} - \frac{20}{4}$$

$$R(x) = \left( x + \frac{5}{2} \right)^2 + \frac{-45}{4}$$

### Corrigé de l'exercice 6

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = 16x^2 - 72x + 81$$

$$= (4x - 9)^2$$

$$= \left( 4 \times \left( x - \frac{9}{4} \right) \right)^2$$

$$P(x) = 16 \times \left( x - \frac{9}{4} \right)^2$$

$$Q(x) = x^2 - 11x + 2$$

$$= \left( x - \frac{11}{2} \right)^2 - \left( \frac{11}{2} \right)^2 + 2$$

$$= \left( x - \frac{11}{2} \right)^2 + \frac{-121}{4} + \frac{2 \times 4}{1 \times 4}$$

$$= \left( x - \frac{11}{2} \right)^2 + \frac{-121}{4} + \frac{8}{4}$$

$$Q(x) = \left( x - \frac{11}{2} \right)^2 + \frac{-113}{4}$$

$$S(x) = 3x^2 + 2x - 5$$

$$= 3 \times \left( x^2 + \frac{2}{3}x - \frac{5}{3} \right)$$

$$= 3 \times \left( \left( x + \frac{1}{3} \right)^2 - \left( \frac{1}{3} \right)^2 + \frac{-5}{3} \right)$$

$$= 3 \times \left( \left( x + \frac{1}{3} \right)^2 + \frac{-1}{9} + \frac{-5 \times 3}{3 \times 3} \right)$$

$$= 3 \times \left( \left( x + \frac{1}{3} \right)^2 + \frac{-1}{9} + \frac{-15}{9} \right)$$

$$= 3 \times \left( \left( x + \frac{1}{3} \right)^2 + \frac{-16}{9} \right)$$

$$= 3 \times \left( x + \frac{1}{3} \right)^2 + \frac{-16 \times 3}{3 \times 3}$$

$$S(x) = 3 \times \left( x + \frac{1}{3} \right)^2 + \frac{-16}{3}$$

$$R(x) = x^2 + 12x + 6$$

$$= (x + 6)^2 - 6^2 + 6$$

$$= (x + 6)^2 - 36 + 6$$

$$R(x) = (x + 6)^2 - 30$$

### Corrigé de l'exercice 7

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = x^2 + 5x + 2$$

$$= \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 2$$

$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{2 \times 4}{1 \times 4}$$

$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{8}{4}$$

$$\boxed{P(x) = \left(x + \frac{5}{2}\right)^2 + \frac{-17}{4}}$$

$$Q(x) = 4x^2 + 16x + 16$$

$$= (2x + 4)^2$$

$$= \left(2 \times \left(x + \frac{4}{2}\right)\right)^2$$

$$\boxed{Q(x) = 4 \times (x + 2)^2}$$

$$S(x) = x^2 + 16x + 9$$

$$= (x + 8)^2 - 8^2 + 9$$

$$= (x + 8)^2 - 64 + 9$$

$$\boxed{S(x) = (x + 8)^2 - 55}$$

$$R(x) = -3x^2 - 2x + 6$$

$$= -3 \times \left(x^2 + \frac{2}{3}x - 2\right)$$

$$= -3 \times \left(\left(x + \frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 - 2\right)$$

$$= -3 \times \left(\left(x + \frac{1}{3}\right)^2 + \frac{-1}{9} - \frac{2 \times 9}{1 \times 9}\right)$$

$$= -3 \times \left(\left(x + \frac{1}{3}\right)^2 + \frac{-1}{9} - \frac{18}{9}\right)$$

$$= -3 \times \left(\left(x + \frac{1}{3}\right)^2 + \frac{-19}{9}\right)$$

$$= -3 \times \left(x + \frac{1}{3}\right)^2 + \frac{-19 \times \cancel{3} \times (-1)}{\cancel{3} \times 3}$$

$$\boxed{R(x) = -3 \times \left(x + \frac{1}{3}\right)^2 + \frac{19}{3}}$$

### Corrigé de l'exercice 8

Donner la forme canonique des polynômes  $P$ ,  $Q$ ,  $R$  et  $S$ .

$$P(x) = -4x^2 - 9x + 8$$

$$= -4 \times \left(x^2 + \frac{9}{4}x - 2\right)$$

$$= -4 \times \left(\left(x + \frac{9}{8}\right)^2 - \left(\frac{9}{8}\right)^2 - 2\right)$$

$$= -4 \times \left(\left(x + \frac{9}{8}\right)^2 + \frac{-81}{64} - \frac{2 \times 64}{1 \times 64}\right)$$

$$= -4 \times \left(\left(x + \frac{9}{8}\right)^2 + \frac{-81}{64} - \frac{128}{64}\right)$$

$$= -4 \times \left(\left(x + \frac{9}{8}\right)^2 + \frac{-209}{64}\right)$$

$$= -4 \times \left(x + \frac{9}{8}\right)^2 + \frac{-209 \times \cancel{4} \times (-1)}{\cancel{4} \times 16}$$

$$\boxed{P(x) = -4 \times \left(x + \frac{9}{8}\right)^2 + \frac{209}{16}}$$

$$Q(x) = x^2 - 3x - 9$$

$$= \left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 9$$

$$= \left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{9 \times 4}{1 \times 4}$$

$$= \left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{36}{4}$$

$$\boxed{Q(x) = \left(x - \frac{3}{2}\right)^2 + \frac{-45}{4}}$$

$$R(x) = x^2 + 18x + 8$$

$$= (x + 9)^2 - 9^2 + 8$$

$$= (x + 9)^2 - 81 + 8$$

$$\boxed{R(x) = (x + 9)^2 - 73}$$

$$S(x) = 4x^2 + 36x + 81$$

$$= (2x + 9)^2$$

$$= \left(2 \times \left(x + \frac{9}{2}\right)\right)^2$$

$$\boxed{S(x) = 4 \times \left(x + \frac{9}{2}\right)^2}$$