

Corrigé de l'exercice 1Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = -5x^2 + 2x - 3$$

$$= -5 \times \left(x^2 - \frac{2}{5}x + \frac{3}{5} \right)$$

$$= -5 \times \left(\left(x - \frac{1}{5} \right)^2 - \left(\frac{1}{5} \right)^2 + \frac{3}{5} \right)$$

$$= -5 \times \left(\left(x - \frac{1}{5} \right)^2 + \frac{-1}{25} + \frac{3 \times 5}{5 \times 5} \right)$$

$$= -5 \times \left(\left(x - \frac{1}{5} \right)^2 + \frac{-1}{25} + \frac{15}{25} \right)$$

$$= -5 \times \left(\left(x - \frac{1}{5} \right)^2 + \frac{14}{25} \right)$$

$$= -5 \times \left(x - \frac{1}{5} \right)^2 + \frac{14 \times 5 \times (-1)}{5 \times 5}$$

$$P(x) = -5 \times \left(x - \frac{1}{5} \right)^2 + \frac{-14}{5}$$

$$Q(x) = x^2 - 8x + 8$$

$$= (x - 4)^2 - 4^2 + 8$$

$$= (x - 4)^2 - 16 + 8$$

$$Q(x) = (x - 4)^2 - 8$$

$$S(x) = x^2 + 12x + 36$$

$$S(x) = (x + 6)^2$$

$$R(x) = x^2 - 9x + 1$$

$$= \left(x - \frac{9}{2} \right)^2 - \left(\frac{9}{2} \right)^2 + 1$$

$$= \left(x - \frac{9}{2} \right)^2 + \frac{-81}{4} + \frac{1 \times 4}{1 \times 4}$$

$$= \left(x - \frac{9}{2} \right)^2 + \frac{-81}{4} + \frac{4}{4}$$

$$R(x) = \left(x - \frac{9}{2} \right)^2 + \frac{-77}{4}$$

Corrigé de l'exercice 2Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 - 4x + 6$$

$$= (x - 2)^2 - 2^2 + 6$$

$$= (x - 2)^2 - 4 + 6$$

$$P(x) = (x - 2)^2 + 2$$

$$Q(x) = 5x^2 + 4x - 6$$

$$= 5 \times \left(x^2 + \frac{4}{5}x - \frac{6}{5} \right)$$

$$= 5 \times \left(\left(x + \frac{2}{5} \right)^2 - \left(\frac{2}{5} \right)^2 + \frac{-6}{5} \right)$$

$$= 5 \times \left(\left(x + \frac{2}{5} \right)^2 + \frac{-4}{25} + \frac{-6 \times 5}{5 \times 5} \right)$$

$$= 5 \times \left(\left(x + \frac{2}{5} \right)^2 + \frac{-4}{25} + \frac{-30}{25} \right)$$

$$= 5 \times \left(\left(x + \frac{2}{5} \right)^2 + \frac{-34}{25} \right)$$

$$= 5 \times \left(x + \frac{2}{5} \right)^2 + \frac{-34 \times 5}{5 \times 5}$$

$$R(x) = 9x^2 - 6x + 1$$

$$= (3x - 1)^2$$

$$= \left(3 \times \left(x - \frac{1}{3} \right) \right)^2$$

$$R(x) = 9 \times \left(x - \frac{1}{3} \right)^2$$

$$Q(x) = 5 \times \left(x + \frac{2}{5} \right)^2 + \frac{-34}{5}$$

$$S(x) = x^2 + 11x - 4$$

$$= \left(x + \frac{11}{2} \right)^2 - \left(\frac{11}{2} \right)^2 - 4$$

$$= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} - \frac{4 \times 4}{1 \times 4}$$

$$= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} - \frac{16}{4}$$

$$S(x) = \left(x + \frac{11}{2} \right)^2 + \frac{-137}{4}$$

Corrigé de l'exercice 3

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 4x^2 - 16x + 16$$

$$= (2x - 4)^2$$

$$= \left(2 \times \left(x - \frac{4}{2}\right)\right)^2$$

$$\boxed{P(x) = 4 \times (x - 2)^2}$$

$$S(x) = x^2 + 6x + 4$$

$$= (x + 3)^2 - 3^2 + 4$$

$$= (x + 3)^2 - 9 + 4$$

$$\boxed{S(x) = (x + 3)^2 - 5}$$

$$Q(x) = -5x^2 + 3x - 1$$

$$= -5 \times \left(x^2 - \frac{3}{5}x + \frac{1}{5}\right)$$

$$= -5 \times \left(\left(x - \frac{3}{10}\right)^2 - \left(\frac{3}{10}\right)^2 + \frac{1}{5}\right)$$

$$= -5 \times \left(\left(x - \frac{3}{10}\right)^2 + \frac{-9}{100} + \frac{1 \times 20}{5 \times 20}\right)$$

$$= -5 \times \left(\left(x - \frac{3}{10}\right)^2 + \frac{-9}{100} + \frac{20}{100}\right)$$

$$= -5 \times \left(\left(x - \frac{3}{10}\right)^2 + \frac{11}{100}\right)$$

$$= -5 \times \left(x - \frac{3}{10}\right)^2 + \frac{11 \times 5 \times (-1)}{5 \times 20}$$

$$\boxed{Q(x) = -5 \times \left(x - \frac{3}{10}\right)^2 + \frac{-11}{20}}$$

$$R(x) = x^2 + 7x + 4$$

$$= \left(x + \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 4$$

$$= \left(x + \frac{7}{2}\right)^2 + \frac{-49}{4} + \frac{4 \times 4}{1 \times 4}$$

$$= \left(x + \frac{7}{2}\right)^2 + \frac{-49}{4} + \frac{16}{4}$$

$$\boxed{R(x) = \left(x + \frac{7}{2}\right)^2 + \frac{-33}{4}}$$

Corrigé de l'exercice 4

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 - 11x + 5$$

$$= \left(x - \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 + 5$$

$$= \left(x - \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{5 \times 4}{1 \times 4}$$

$$= \left(x - \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{20}{4}$$

$$\boxed{P(x) = \left(x - \frac{11}{2}\right)^2 + \frac{-101}{4}}$$

$$Q(x) = x^2 - 10x + 5$$

$$= (x - 5)^2 - 5^2 + 5$$

$$= (x - 5)^2 - 25 + 5$$

$$\boxed{Q(x) = (x - 5)^2 - 20}$$

$$S(x) = 16x^2 - 24x + 9$$

$$= (4x - 3)^2$$

$$= \left(4 \times \left(x - \frac{3}{4}\right)\right)^2$$

$$\boxed{S(x) = 16 \times \left(x - \frac{3}{4}\right)^2}$$

$$R(x) = -3x^2 - 6x - 3$$

$$= -3 \times (x^2 + 2x + 1)$$

$$= -3 \times ((x + 1)^2 - 1^2 + 1)$$

$$= -3 \times ((x + 1)^2 - 1 + 1)$$

$$= -3 \times ((x + 1)^2 + 0)$$

$$\boxed{R(x) = -3 \times (x + 1)^2 + 0}$$

Corrigé de l'exercice 5

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 - 8x - 7 \\ &= (x - 4)^2 - 4^2 - 7 \\ &= (x - 4)^2 - 16 - 7 \end{aligned}$$

$$\boxed{P(x) = (x - 4)^2 - 23}$$

$$\begin{aligned} S(x) &= 16x^2 + 32x + 16 \\ &= (4x + 4)^2 \\ &= \left(4 \times \left(x + \frac{4}{4}\right)\right)^2 \end{aligned}$$

$$\boxed{S(x) = 16 \times (x + 1)^2}$$

$$\begin{aligned} Q(x) &= x^2 + 9x + 8 \\ &= \left(x + \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 + 8 \\ &= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} + \frac{8 \times 4}{1 \times 4} \\ &= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} + \frac{32}{4} \end{aligned}$$

$$\boxed{Q(x) = \left(x + \frac{9}{2}\right)^2 + \frac{-49}{4}}$$

$$\begin{aligned} R(x) &= -2x^2 + 2x - 8 \\ &= -2 \times (x^2 - x + 4) \\ &= -2 \times \left(\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 4\right) \\ &= -2 \times \left(\left(x - \frac{1}{2}\right)^2 + \frac{-1}{4} + \frac{4 \times 4}{1 \times 4}\right) \\ &= -2 \times \left(\left(x - \frac{1}{2}\right)^2 + \frac{-1}{4} + \frac{16}{4}\right) \\ &= -2 \times \left(\left(x - \frac{1}{2}\right)^2 + \frac{15}{4}\right) \\ &= -2 \times \left(x - \frac{1}{2}\right)^2 + \frac{15 \times 2 \times (-1)}{2 \times 2} \end{aligned}$$

$$\boxed{R(x) = -2 \times \left(x - \frac{1}{2}\right)^2 + \frac{-15}{2}}$$

Corrigé de l'exercice 6

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= -5x^2 - 8x - 3 \\ &= -5 \times \left(x^2 + \frac{8}{5}x + \frac{3}{5}\right) \\ &= -5 \times \left(\left(x + \frac{4}{5}\right)^2 - \left(\frac{4}{5}\right)^2 + \frac{3}{5}\right) \\ &= -5 \times \left(\left(x + \frac{4}{5}\right)^2 + \frac{-16}{25} + \frac{3 \times 5}{5 \times 5}\right) \\ &= -5 \times \left(\left(x + \frac{4}{5}\right)^2 + \frac{-16}{25} + \frac{15}{25}\right) \\ &= -5 \times \left(\left(x + \frac{4}{5}\right)^2 + \frac{-1}{25}\right) \\ &= -5 \times \left(x + \frac{4}{5}\right)^2 + \frac{-1 \times 5 \times (-1) \times 1}{5 \times 5} \end{aligned}$$

$$\boxed{P(x) = -5 \times \left(x + \frac{4}{5}\right)^2 + \frac{1}{5}}$$

$$\begin{aligned} Q(x) &= 36x^2 + 108x + 81 \\ &= (6x + 9)^2 \\ &= \left(6 \times \left(x + \frac{9}{6}\right)\right)^2 \end{aligned}$$

$$\boxed{Q(x) = 36 \times \left(x + \frac{3}{2}\right)^2}$$

$$\begin{aligned} S(x) &= x^2 + 6x - 6 \\ &= (x + 3)^2 - 3^2 - 6 \\ &= (x + 3)^2 - 9 - 6 \end{aligned}$$

$$\boxed{S(x) = (x + 3)^2 - 15}$$

$$\begin{aligned} R(x) &= x^2 + 3x - 7 \\ &= \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 7 \\ &= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{7 \times 4}{1 \times 4} \\ &= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{28}{4} \end{aligned}$$

$$\boxed{R(x) = \left(x + \frac{3}{2}\right)^2 + \frac{-37}{4}}$$

Corrigé de l'exercice 7

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = -5x^2 - x + 6$$

$$= -5 \times \left(x^2 + \frac{1}{5}x - \frac{6}{5} \right)$$

$$= -5 \times \left(\left(x + \frac{1}{10} \right)^2 - \left(\frac{1}{10} \right)^2 + \frac{-6}{5} \right)$$

$$= -5 \times \left(\left(x + \frac{1}{10} \right)^2 + \frac{-1}{100} + \frac{-6 \times 20}{5 \times 20} \right)$$

$$= -5 \times \left(\left(x + \frac{1}{10} \right)^2 + \frac{-1}{100} + \frac{-120}{100} \right)$$

$$= -5 \times \left(\left(x + \frac{1}{10} \right)^2 + \frac{-121}{100} \right)$$

$$= -5 \times \left(x + \frac{1}{10} \right)^2 + \frac{-121 \times 5 \times (-1)}{5 \times 20}$$

$$P(x) = -5 \times \left(x + \frac{1}{10} \right)^2 + \frac{121}{20}$$

$$Q(x) = x^2 - 4x - 8$$

$$= (x - 2)^2 - 2^2 - 8$$

$$= (x - 2)^2 - 4 - 8$$

$$Q(x) = (x - 2)^2 - 12$$

$$S(x) = 81x^2 + 54x + 9$$

$$= (9x + 3)^2$$

$$= \left(9 \times \left(x + \frac{3}{9} \right) \right)^2$$

$$S(x) = 81 \times \left(x + \frac{1}{3} \right)^2$$

$$R(x) = x^2 + 9x + 9$$

$$= \left(x + \frac{9}{2} \right)^2 - \left(\frac{9}{2} \right)^2 + 9$$

$$= \left(x + \frac{9}{2} \right)^2 + \frac{-81}{4} + \frac{9 \times 4}{1 \times 4}$$

$$= \left(x + \frac{9}{2} \right)^2 + \frac{-81}{4} + \frac{36}{4}$$

$$R(x) = \left(x + \frac{9}{2} \right)^2 + \frac{-45}{4}$$

Corrigé de l'exercice 8

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 4x^2 - 9x - 3$$

$$= 4 \times \left(x^2 - \frac{9}{4}x - \frac{3}{4} \right)$$

$$= 4 \times \left(\left(x - \frac{9}{8} \right)^2 - \left(\frac{9}{8} \right)^2 + \frac{-3}{4} \right)$$

$$= 4 \times \left(\left(x - \frac{9}{8} \right)^2 + \frac{-81}{64} + \frac{-3 \times 16}{4 \times 16} \right)$$

$$= 4 \times \left(\left(x - \frac{9}{8} \right)^2 + \frac{-81}{64} + \frac{-48}{64} \right)$$

$$= 4 \times \left(\left(x - \frac{9}{8} \right)^2 + \frac{-129}{64} \right)$$

$$= 4 \times \left(x - \frac{9}{8} \right)^2 + \frac{-129 \times 4}{4 \times 16}$$

$$P(x) = 4 \times \left(x - \frac{9}{8} \right)^2 + \frac{-129}{16}$$

$$Q(x) = 36x^2 - 108x + 81$$

$$= (6x - 9)^2$$

$$= \left(6 \times \left(x - \frac{9}{6} \right) \right)^2$$

$$Q(x) = 36 \times \left(x - \frac{3}{2} \right)^2$$

$$R(x) = x^2 + 8x + 3$$

$$= (x + 4)^2 - 4^2 + 3$$

$$= (x + 4)^2 - 16 + 3$$

$$R(x) = (x + 4)^2 - 13$$

$$S(x) = x^2 + 5x - 3$$

$$= \left(x + \frac{5}{2} \right)^2 - \left(\frac{5}{2} \right)^2 - 3$$

$$= \left(x + \frac{5}{2} \right)^2 + \frac{-25}{4} - \frac{3 \times 4}{1 \times 4}$$

$$= \left(x + \frac{5}{2} \right)^2 + \frac{-25}{4} - \frac{12}{4}$$

$$S(x) = \left(x + \frac{5}{2} \right)^2 + \frac{-37}{4}$$