Corrigé de l'exercice 1

On utilise les relations :

$$P(A \cap B) = P(A)\,P(B \mid A), \qquad P(\overline{A} \cap B) = P(\overline{A})\,P(B \mid \overline{A})$$

et

$$P(B) = P(A \cap B) + P(\overline{A} \cap B), \qquad P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

Redaction attendue.

- On calcule $P(A \cap B) = P(A)P(B \mid A)$ et $P(\overline{A} \cap B) = P(\overline{A})P(B \mid \overline{A})$.
- On en deduit $P(B) = P(A \cap B) + P(\overline{A} \cap B)$.
- Comme $P(B) \neq 0$, on calcule $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.
- On donne une valeur approchee au millieme.

$$\begin{aligned}
P(\overline{A}) = 1 - P(A) = 1 - \frac{9}{50} &= \frac{41}{50} \\
P(A \cap B) = P(A)\,P(B \mid A) = \frac{9}{50} \times \frac{3}{4} &= \frac{27}{200} \\
P(\overline{A} \cap B) = P(\overline{A})\,P(B \mid \overline{A}) = \frac{41}{50} \times \frac{3}{100} &= \frac{123}{5000} \\
P(B) = P(A \cap B) + P(\overline{A} \cap B) = \frac{27}{200} + \frac{123}{5000} &= \frac{399}{2500} \\
P(B) &\approx 0.160 \\
P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{27}{200}}{\frac{399}{2500}} &= \frac{225}{266} \\
P(A \mid B) &\approx 0.846
\end{aligned}$$

Interpretation : Ceci represente la probabilite que l'eleve connaisse la reponse sachant qu'il a repondu juste.

Corrigé de l'exercice 2

On utilise les relations :

$$P(A \cap B) = P(A)\,P(B \mid A), \qquad P(\overline{A} \cap B) = P(\overline{A})\,P(B \mid \overline{A})$$

et

$$P(B) = P(A \cap B) + P(\overline{A} \cap B), \qquad P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

Redaction attendue.

- On calcule $P(A \cap B) = P(A)P(B \mid A)$ et $P(\overline{A} \cap B) = P(\overline{A})P(B \mid \overline{A})$.
- On en deduit $P(B) = P(A \cap B) + P(\overline{A} \cap B)$.
- Comme $P(B) \neq 0$, on calcule $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.
- On donne une valeur approchee au millieme.

$$
\begin{aligned}
P(\overline{A}) &= 1 - P(A) = 1 - \frac{3}{20} = \frac{17}{20}\\
P(A \cap B) &= P(A)\,P(B \mid A) = \frac{3}{20} \times \frac{19}{20} = \frac{57}{400}\\
P(\overline{A} \cap B) &= P(\overline{A})\,P(B \mid \overline{A}) = \frac{17}{20} \times \frac{3}{100} = \frac{51}{2000}\\
P(B) &= P(A \cap B) + P(\overline{A} \cap B) = \frac{57}{400} + \frac{51}{2000} = \frac{21}{125}\\
P(B) &\approx 0.168\\
P(A \mid B) &= \frac{P(A \cap B)}{P(B)} = \frac{\frac{57}{400}}{\frac{21}{125}} = \frac{95}{112}\\
P(A \mid B) &\approx 0.848
\end{aligned}
$$

Interpretation : Ceci represente la probabilite qu'une personne soit malade sachant que son test est positif.

Corrigé de l'exercice 3

On utilise les relations :

$$P(A \cap B) = P(A)\,P(B \mid A), \qquad P(\overline{A} \cap B) = P(\overline{A})\,P(B \mid \overline{A})$$

et

$$P(B) = P(A \cap B) + P(\overline{A} \cap B), \qquad P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

Redaction attendue.

- On applique la formule des probabilites totales : $P(B) = P(A)P(B \mid A) + P(\overline{A})P(B \mid \overline{A})$.
- On calcule ensuite $P(A \cap B) = P(A)P(B \mid A)$.
- Comme $P(B) \neq 0$, on calcule $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.
- On donne une valeur approchee et on interprete le resultat dans le contexte.

$$
\begin{aligned}
P(\overline{A}) &= 1 - P(A) = 1 - \frac{1}{5} = \frac{4}{5}\\
P(A \cap B) &= P(A)\,P(B \mid A) = \frac{1}{5} \times \frac{17}{20} = \frac{17}{100}\\
P(\overline{A} \cap B) &= P(\overline{A})\,P(B \mid \overline{A}) = \frac{4}{5} \times \frac{1}{20} = \frac{1}{25}\\
P(B) &= P(A \cap B) + P(\overline{A} \cap B) = \frac{17}{100} + \frac{1}{25} = \frac{21}{100}\\
P(B) &\approx 0.210\\
P(A \mid B) &= \frac{P(A \cap B)}{P(B)} = \frac{\frac{17}{100}}{\frac{21}{100}} = \frac{17}{21}\\
P(A \mid B) &\approx 0.810
\end{aligned}
$$

Interpretation : Ceci represente la probabilite que le serveur soit sature sachant qu'une connexion a echoue.